



# Using Vector Fields in the Modelling of Movements as Flows

## A Case Study with Cattle Trade Networks

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**Abstract.** Livestock production is one of the world's most important economic activities, involving nearly every country either as a producer or consumer. Indeed, around 1.3 billion people worldwide depend on livestock production. As livestock numbers increase due to the growth in the world population, so does the need for modelling and understanding the patterns in the movement of livestock, which is crucial for understanding global epidemic patterns. Interestingly, the structure of livestock movement is quite similar to other movements, such as human mobility, if we model the phenomena as cases of origin-destination (O-D) flows. Here, we introduce a methodology to better understand the dynamics of mobility patterns by characterising them as these flows while accounting for spatial information. Our approach looks into flows of movements as something that can be derived from networks. We demonstrate the power of our approach on cattle trading by examining a dataset from the Brazilian state of Minas Gerais, the country's largest cattle production. Our proposal is general and fits to any case in which the network is build from an O-D matrix.

**Keywords:** Origin-destination networks · Flow maps · Vector fields · Cattle epidemic modelling · Cattle trade networks

## 1 Introduction

Animal production is a major component of economies worldwide. It is accepted that animal production support 1 of every 5 people in the world [28], which is a demand that continues to increase as the planet's human population grows. 71 million tones of beef products were produced in 2018 by the top-6 producers in the world, namely, USA, Brazil, China, India, Argentina, and Australia. In 2020, the projected number of bovines in Latin America and the Caribbean was 0.5 billion and in North America and Europe was 0.3 billion [27].

These large markets may suffer catastrophic losses when affected by contagious diseases. In 2021, the population of cattle and calves in the United Kingdom was approximately 9.44 million heads. However, this number was supposed to be larger because a foot-and-mouth (FMD) epidemic in 2001 resulted in the slaughter of approximately 3 million animals. The epidemic costed the UK agriculture industry around £3.1 billion but also affected other major sectors of the economy, such as the tourism industry, which suffered losses ranging from £140 million to £500 million a week [24].

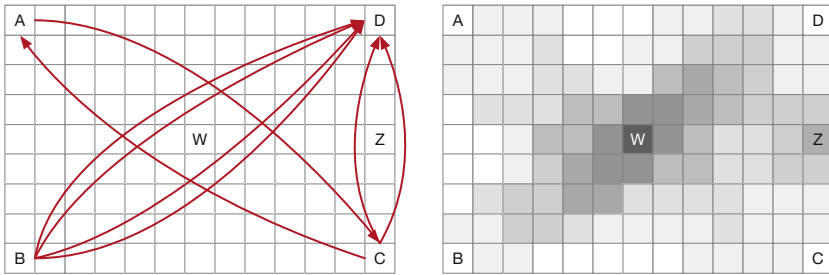
To improve their disease-monitoring capabilities, many countries have invested in tools for tracking animal production, creating a market estimated at US\$ 24 billion. For example, Australia established a livestock traceability system based on electronic ear tags for cattle management. Thailand tracks the inter-provincial movements of livestock by an online system called *e-movement* [33]. And since 2001, Japan has implemented a food card system based on the agricultural biographical system, which tracks agricultural production and marketing [35].

While current monitoring tools can provide an overview of important aspects of the animal product market, they might miss signs of animal diseases that can be hard to quantify because the losses might come from hidden sources. For example, while visible sources include the death of animals, decreased access to food, and poor quality of animal products, hidden losses might occur from a change in animal population structure, increased labour costs, and environmental issues such as CO<sub>2</sub> emissions.

There are many Network-Science approaches to model epidemics in various regions [3, 11, 17, 19, 21] using a variety of epidemiological approaches [16, 23, 34]. While the power of network approaches to model disease spread is undeniable, they suffer from shortcomings. Very few approaches have used large datasets such as the ones available in Brazil or the USA [12] where the level of uncertainty about the trade is quite high [2]. For example, Brazil is the second-largest producer of beef in the world with an annual production of 10.3 million metric tons, behind only the USA with 12.5 million metric tons and ahead of the European Union with 7.8 million tons [1]. Yet, as mentioned above, Brazil is known to have a less-than-perfect tracking system leading to uncertainty in the trade network [2].

Network-based approaches often ignore relevant spatial information; when cattle herds are transported between locations, areas in-between are affected. This information is lost in networks because they only capture origin-destination pairs. If the locations between the origins and destinations are not part of any trade, they are ignored in the network representation. However, they might have higher spatial centrality because ground transportation is used in most cattle movements. Figure 1 shows an example in which locations A, B, C, and D are origins and destinations, but W and Z are not. In the network representation (on the left), W and Z are not even nodes, and a network analysis would overlook that the flow of cattle passes through those locations (e.g., the shortest-paths

movements). In the representation on the right, we can see the betweenness of the locations in which W and Z have central positions in the spatial movements.



**Fig. 1. Network modelling might miss important locations.** In this toy example we have several movements represented as edges in a spatial network:  $(A \rightarrow C)$ ,  $(C \rightarrow A)$ ,  $4 \times (B \rightarrow D)$ , and  $2 \times (C \rightarrow D)$ . On the network representation on the left, the locations W and Z are not part of the representation because they are not involved in movements. However, the representation on the right has the potential to point out that locations W and Z are important as a confluence of spatial movements from several locations.

Hence, we propose to convert the network of movements into a vector-field representation, as it complements the benefits of network analyses. Furthermore, when datasets lack information (e.g., noise, uncertainty), our approach is more intuitive in adding locations based on vector fields than adding a network node or link. Last, our proposal is less computationally intensive than network algorithms, in particular, algorithms to analyse the dynamics of the network. Our approach is tested on a very large dataset from Brazil. We focus on cattle trading and examine a dataset from the Brazilian state of Minas Gerais, the country's largest cattle production. Our results show that our model provides an adequate representation that allows us to see the dynamics of trade and the effect of uncertainty even when datasets are massive.

## 2 Related Works

The understanding of cattle trade and its modelling using network science has caught the attention of researchers, given that networks are an excellent framework for capturing the structure of connections and cattle trade is primarily an origin-destination phenomenon in which cattle herds are moved around for various purposes: sales, fattening, slaughtering, to name a few. Networks can capture the structure of this economic activity, which in turn can be used to predict possible epidemic behaviours such as FMD disease.

Several works appeared in the literature after the FMD outbreak of 2001 when issues related to managing and controlling of infectious diseases in livestock were raised [5, 13, 14]. These works employed a network modelling that traces the

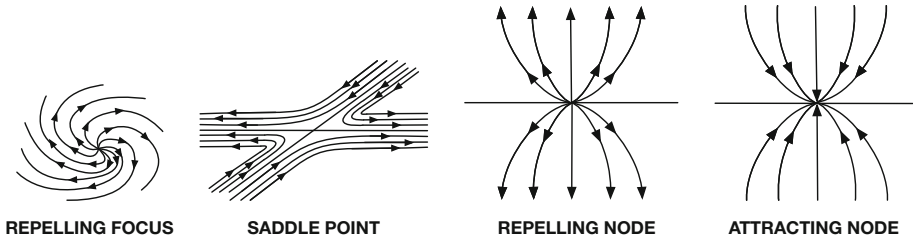
transmission of diseases using the contacts between livestock locations that arise from trade activities.

After 2005, the use of the techniques from network science became more commonplace, with many works making use of open datasets of livestock movement [3, 7, 20–22, 25]. Generally, the papers agreed with the fact that movement and the contacts are the main reasons for disease spread in livestock, especially cattle. As cattle production keeps growing it becomes necessary to understand the characteristics of livestock contact networks under conditions in the production countries. Most of the work done thus far looks at datasets in countries where the conditions for production are close to ideal, and the tracing can be easily done.

Our proposal complements the use of networks because it can capture dynamic mechanisms that would be hard to do using networks. Our approach can also be done computationally cheaper; hence, it is appropriate for large-scale datasets. We believe this is the first time this approach is used even though the literature mentions of flow field being used in migration patterns; these fields allow us to see various features in the fields such as saddle points, attracting areas, attracting nodes, and others [15]. For instance, Boyandin et al. [8] introduced a tool called JFlowMap for the analysis of flow maps, in particular the visualisation of such flows, yet, their work comprises of flows as networks and not as vector fields which is what we describe here. Their approach has been used to look at flows of specific kinds of movement, such as bicycles [10], showing that flows can be used for mobility data. Yet, such works are essentially network modelling representing the flows. We argue that vector fields are generic and that the methodology used here could very well be used in other spatial mobility data, such as human mobility [4].

In this paper, we use vector fields to model cattle trade flows. Our goal is to apply vector fields to other mobility data, but, for this, we have to use interpolation to make sure the field has a complete representation of the flow. Many tools exist to visualise vector fields, and a good comparison is provided by Laidlaw et al. [18]. Vector fields have been extensively used in weather, as they also provide better visualisations of the dynamics of such systems [30]. One of the main advantages is that one can use approaches for the identification of unusual/critical structures in these vector fields.

Figure 2 shows a few examples of critical structures that can be given a semantic interpretation in the context of epidemics and cattle movement. For instance, the “repelling focus” and “repelling node” structures can be considered possible sources of an epidemic, while an “attracting node” could be seen as the sink of an epidemic. That means that sources are rarely the destination of an outbreak. Vector fields allow for easier identification and visualisation of these substructures and how they change over time; the dynamics of sinks and sources can drive different interventions in the case of an epidemic; the intervention can be designed to deal with seasonality of movements if the vector flow reveals such patterns.



**Fig. 2. Example of critical structures in vector fields.** The use of vector fields allows for the identification of spatial critical points such as the ones in this figure. We can also look at the dynamics of these points overtime.

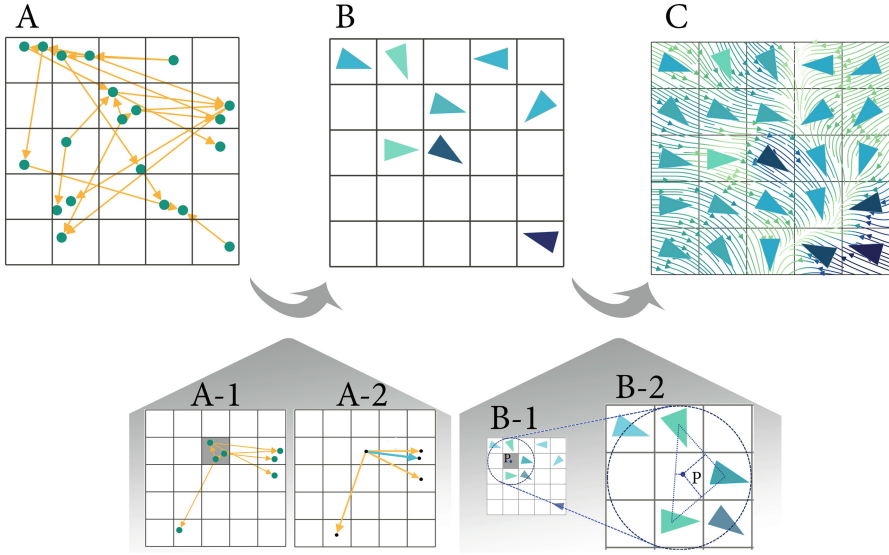
### 3 Methodology: Mobility Vector Fields

#### 3.1 From Networks to Vector Fields

We indicated earlier that we want to go from an origin-destination representation (a network) to a vector field representation for the region being studied. Hence, the first step is to set divisions of the region being investigated—a granularity of the study—and project the network on this space. With this step, network nodes will be part of a particular cell in the spatial representation (Fig. 3A). With the projection in Fig. 3A, we can look into all the outgoing edges for each cell and taking them as vectors with distance and value, and then combine them into one vector joining pairs of cells. This is illustrated in Fig. 3A-1 and Fig. 3A-2 for one cell shown in grey. The set of vectors for the grey cell is combined in a way that only one vector exists between any pair of cells (yellow vectors in Fig. 3A-2). The second step is then to calculate the resulting vector from all the yellow vectors outgoing from a particular cell; this resulting vector shown in green in Fig. 3A-2 represents the general flow direction for that cell. Once this process is done for all cells with nodes with outgoing edges, we end up with what is shown in Fig. 3B. Note that only cells that have network nodes with outgoing edges have vectors, which means the vector field is incomplete. We will use an interpolation method to complete the field.

#### 3.2 Vector Field Interpolation

As we have seen in the steps described in the previous section, the conversion of a network into a vector field, leads to a field where many locations in the studied area may not have a vector because they were not origins of for edges in the network (Fig. 3B). To assess the global behaviour of the vector field patterns, we need to complete the field with values that represent the flow in that location. In fact, in order to have an actual idea of flow, we have to interpolate in other points also. Interpolation methods involve constructing vectors for new points based on already-known vectors.



**Fig. 3. The process of generating a vector field from a network.** (A) a directed network representing movement is projected on a spatial map divided into parts. (A-1) focus of one cell/part of the space and the edges outgoing from that cell (in grey). (A-2) the movements are then converted into a resulting vector with the starting point at the centre of the origin cell (grey cell) and the end point at the centre of the destination cell (shown as the green vector). (B) all the resulting vectors from the network in (A) showing that many cells do not have a vector. (B-1) and (B-2) in order to estimate the vectors for each desired point in the grid, we implement a triangle-based interpolation method; the value of vector at a point  $P$  can be obtained by using the three nearest points with vectors. (C) interpolated vector field; different colours indicated different vector sizes.

There are multiple interpolation methods to employ when attempting to develop a continuous vector field. A triangle-based interpolation [31,32] is performed to estimate vector values at points in the grid where their vectors have not been determined. This method uses the known vectors of the three nearest points to calculate the vector for each point in space. As a result, to get the vector of point  $P$  in the grey cell in Fig. 3B-1, the three nearest points having vectors are used as illustrated in Fig. 3B-2. Assume that  $\mathbf{v}_1$ ,  $\mathbf{v}_2$ , and  $\mathbf{v}_3$  are the three nearest vectors.  $h_1$ ,  $h_2$ , and  $h_3$  represent the distance between point  $P$  and the side opposite the angle with the same indexed vector. Using triangle barycentric coordinates [29] and Delaunay triangulation on points with initial vectors, the vector at point  $P$  is determined using Eq. 1.

$$\mathbf{v}_P = \frac{h_1}{h_1 + h_2 + h_3} \mathbf{v}_1 + \frac{h_2}{h_1 + h_2 + h_3} \mathbf{v}_2 + \frac{h_3}{h_1 + h_2 + h_3} \mathbf{v}_3. \quad (1)$$

Once the triangulation is done for every point of interest, we end up with a complete vector field as depicted in Fig. 3C—this figure shows the main flows for each cell but also the flow for the entire field with more points being used.

### 3.3 Dynamics of Fields

We believe that the conversion from networks to vector fields leads to an interesting way of modelling the dynamics of mobility flows. In the case of cattle trade (Sect. 4), this is very useful because it allows us to observe seasonal patterns and locations that deserve more attention as they appear as features in the fields (Fig. 2) but also how these features change overtime.

The exploration of flow patterns globally, requires a concentration on a region (e.g., a part involved in an established division or a particular spatial part of a map) and observing how that region behaves overtime in terms of mobility. The dominant vector direction of a region, the way it changes overtime, and the similarity between the behaviour of vectors of different regions could be easily surveyed through a vector-field representation.

A cosine similarity approach is one possible method that allows us to determine the pattern of the flow direction originating from each region. The cosine similarity factor for each region could be calculated as

$$S_C(\mathbf{v}_k^i, \mathbf{v}_k^j) = \frac{\mathbf{v}_k^i \cdot \mathbf{v}_k^j}{\|\mathbf{v}_k^i\| \|\mathbf{v}_k^j\|}, \quad (2)$$

representing the dynamic auto-correlation of the vector for the region (spatial part)  $k$ ,  $\mathbf{v}_k$ , in the time intervals  $i$  and  $j$ ; this approach is adapted from the concept of dynamic correlation between two regions [6].

The dynamic analysis in the dominant flow directions can provide insightful information for decision makers. For this, flow directions are calculated over a set of pre-determined time periods. For instance, if we choose to use ten time intervals, we can generate ten vector fields using the process depicted in Fig. 3; the result is then the set of fields as seen in Fig. 4A-1.

Afterwards, the cosine value between each pair of consecutive time intervals is calculated using Eq. 2 to examine the changes in the vector related to each area of interest (i.e., a grid cell). In essence, this means that we have a feature vector for each area of interest, allowing us to cluster the areas based on feature-vector similarity. A k-means clustering method is used here; parts in the same group behave more similarly overtime than parts in other groups (Fig. 4A-2).

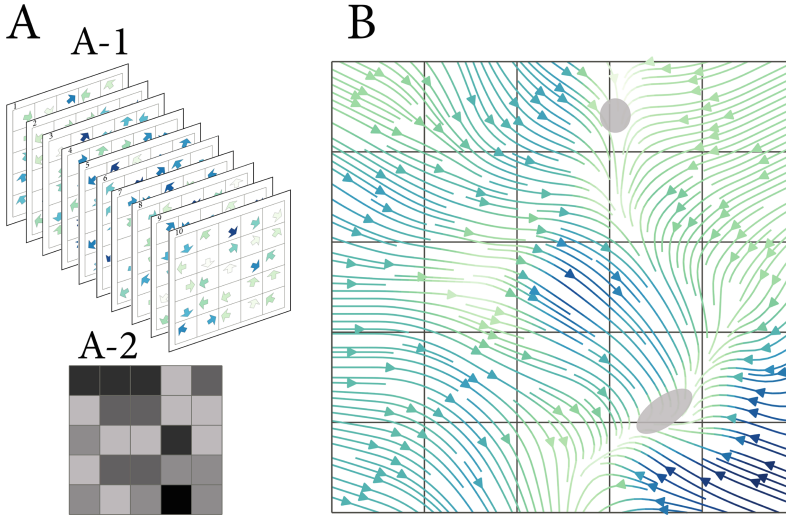
Clustering parts based on their mobility direction overtime helps us determine whether a part has an essential role in the global behaviour of mobility flow.

### 3.4 Identification of Critical Points

The analysis of vector fields can be accomplished by estimating it near some particular location. Vector fields have so-called critical points that reveal insights

into their characteristics when we observe the global behaviour of the field near them. Critical points are points where the flow vanishes. Surrounding these points, the vector field has a distinct structure (Fig. 2).

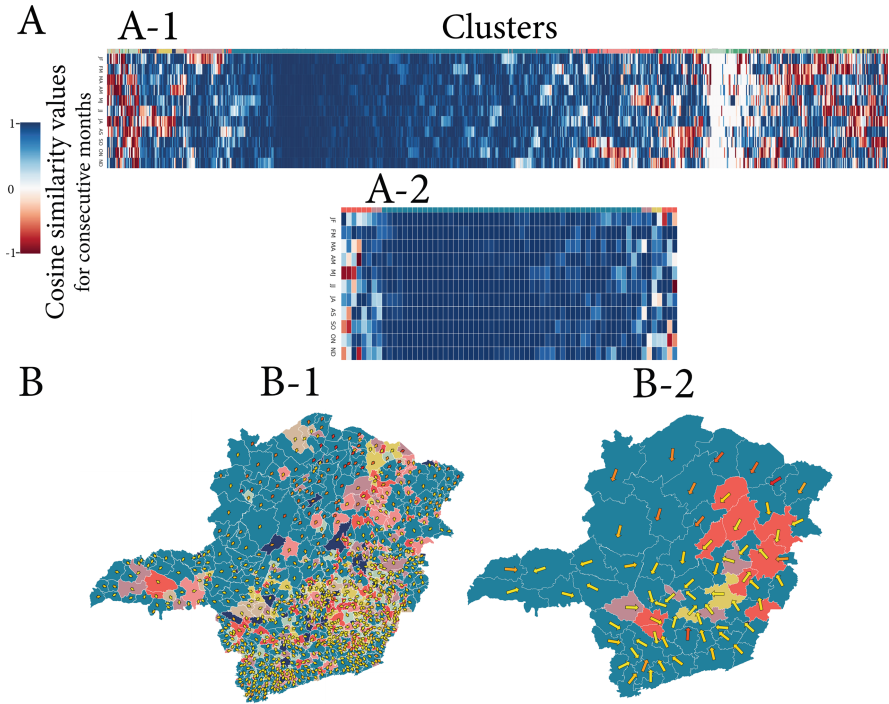
Classification of critical points as sink, source, and saddle points is done according to the sign of the eigenvalues of the Jacobian matrix [26]; which is the matrix of all first-order partial derivatives of the vector field. Figure 4B shows the sink areas in a vector field generated from the network related to one of the time intervals between the nodes in Fig. 3A.



**Fig. 4. Clustering and analysis of critical points in vector fields. (A)** Using cosine similarity to cluster areas based on the dynamics of their vectors overtime. **(A-1)** ten vector fields derived from mobility network for ten time intervals. **(A-2)** Clustering sub-areas according to vector direction over the ten vector fields; colours indicate clusters. **(B)** Vector-field visualisation, with sinks shown in grey.

## 4 Case Study: Cattle Trade

In order to look at the proposed methodology in a real scenario, we use the cattle mobility dataset from the state of Minas Gerais in Brazil [9]. In this dataset, every cattle trade movement is recorded including information about the origin and destination of the movement, the purpose of the trade, the date of the transaction, number of animals moved, and premises identification. This dataset covers a period of four years from 2013 to 2016.



**Fig. 5.** Cosine similarity between vectors of (A-1) cities and (A-2) micro-regions for eleven consecutive months of 2013. As an example, two flow maps of two consecutive months Jan-Feb are used to calculate the similarity of the angle between two vectors associated with each city. The result of the cosine similarity calculation for each city is a set of values ranging from -1 to 1. Using k-means clustering, cities are grouped according to their similarity values (a feature vector). Each group is illustrated with the same colour in (B). (B) Clusters are shown in different colours. The largest cluster of cities (B-1) and micro-regions (B-2) contains the group that does not exhibit a significant change in direction of the vectors.

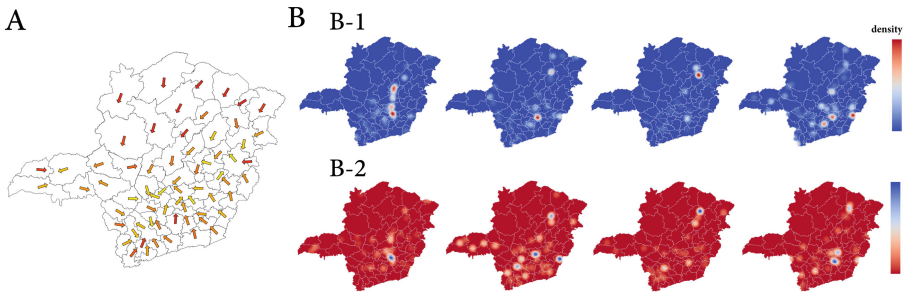
We used the proposed methodology (described in Sect. 3) to cattle trade. Due to space restrictions in this work, we use only one year here (2013) to build networks, generate vector fields, and model trade patterns; results for other years have been generated and will be made available in future works. Vector fields are generated using the monthly trades for two different spatial subdivisions, cities, and micro-regions.

A cosine similarity analysis was conducted for cities as well as micro-regions during the year 2013 in order to demonstrate the month-to-month dynamics. The cosine similarity values for eleven consecutive months (Jan. to Dec.) are shown in Fig. 5A for all cities (Fig. 5A-1) and micro-regions (Fig. 5A-2). The values of the similarity are used to cluster the cities or regions as having similar dynamics within the year; the different colours in Fig. 5B indicate different cluster based on cosine similarity. There is a dominant group in both cases, cities (Fig. 5B-1)

and micro-regions (Fig. 5B-2). Looking at Figure Fig. 5B we can see that several regions have values close to 1 (dark blue). That happens because the vector direction for the cities (or micro-regions) remains nearly unchanged over a year, indicating a predictable direction for cattle trading from those regions.

The cosine similarity measure shows how a city's (or micro-region's) vector behaviour has changed overtime; being in a range from steadiness or dynamic. As well as comparing the behaviour of one spatial area with others, which can lead to the division of larger spatial areas into predictable and unpredictable regions. The next step is to determine whether an area belongs to a critical point in the vector field. In the vector fields, critical points need to be identified for this purpose. Figure 6A illustrates a vector map of micro-regions in the state of Minas Gerais in Brazil while Fig. 6B shows the resulting points of interests, in this case, sinks and sources, for four separate months. One can clearly see that the sinks change for different periods, which indicates that high dynamics of cattle trades in Minas Gerais is present.

When comparing Fig. 5B-2 with Fig. 6B, most of the areas of the largest cluster of micro-regions (those with bluish cyan color in Fig. 5B-2) are not critical points in vector field. Comparatively, the areas with high sink (red areas in Fig. 6B-1) and source (dark blue areas in Fig. 6B-2) density correspond to those that belong to the cluster of parts whose behaviour is not steady. In other words, there are some risky parts in the system that act as sources (or sinks), and we cannot estimate where the epidemic will spread based on their direction of flow. Even though we cannot predict the exact direction in which diseases spread, we can restrict trade by knowing where the source points of flow are for each period



**Fig. 6. (A) Visualisation of a vector field.** The colour of each vector corresponds to the size of the vector. **(B) Critical points in the cattle trade vector field.** Sinks (**B-1**) and sources (**B-2**) are visualised in vector field for four months. Initial vector fields are generated based on trades happening within specific months. A triangle-based interpolation method is used to create a vector field from the initial vectors. We found critical points in this vector field and identified sinks and sources. In (**B-1**), red areas indicate locations with a high density of sinks in the vector fields. Dark blue shows the areas with zero density of sink points. In (**B-2**), the emphasis is on sources. Dark blue areas indicate high-density of sources, and dark red areas are the ones empty of source points.

of time. The next step in future work is to analyse the pattern of changes over time in sinks and sources areas.

## 5 Conclusion and Future Work

We developed an approach that believe can be applied to any type of mobility dataset which contains origins-destinations trajectories. We showed that the move from networks to flows may be beneficial to the understanding of risk areas and points of interest, such as sinks and sources. Our method provides an alternative/complementary tool to network methods for analysing the dynamic patterns of mobility, and it captures the role of intermediate locations reached between origin and destination of moving entities. A strong aspect of this approach is the flexibility in focusing on outgoing or incoming mobility to each part, as well as the availability of many techniques for combining trades originated/destined from/to each part to produce a final vector for that part based on the investigation goal and application. As a result, it becomes more relevant to a particular type of mobility dataset and analysis goals; for instance, one could consider the number of cattle heads being transported as a factor in the value of the vectors.

We have used an example based on cattle movement in Minas Gerais, Brazil to show how the methodology works in real world scenarios. We intend to do the same analysis of flows for incoming edges in the same context but also look at how effective our approach can be in areas such as human mobility [4]. We expect that vector fields can bring more clarity to temporal patterns in human behaviour, particularly in urban environments.

Despite our application to a real dataset, this work would benefit from a confirmation with local authorities in Minas Gerais that the identified critical points indeed correspond to areas in which issues or risks have been noticed in the past. We intend to continue to work with people in Minas Gerais to reach that level and answer questions related to risk such as: are sources indeed found to be locations in which epidemics start? Are sinks found by our method, locations with a higher change of being affected by a disease spread starting at random locations?

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