Using Network Science to Define a Dynamic Communication Topology for Particle Swarm Optimizers

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Abstract. We propose here to use network sciences, specifically an approach based on the Barabási-Albert model, to define a dynamic communication topology for Particle Swarm Optimizers. We compared our proposal to previous approaches, including a simpler Barabási-Albert-based approach and other most used approaches, and we obtained better results in average for well known benchmark functions.

1 Introduction

Particle Swarm Optimization (PSO) is a swarm intelligence technique that has been widely used to solve optimization problems in hyper-dimensional search spaces with continuous variables. PSO was first proposed by Kennedy and Eberhart in 1995 [15] and it was inspired by the social behavior of flocks of birds working together to find food. In the PSO paradigm, each particle in the swarm represents a candidate solution in the fitness function domain. During the algorithm execution, each particle adjusts its velocity and position based on the current position, the current velocity, the best position achieved by itself during the search process so far and the best position obtained by the particles among a pre-determined neighborhood during the search process so far.

There are a few important issues that influence on the convergence velocity and on the quality of the final solution returned at the end of the algorithm execution. Among them are: the equation used to update the velocities of the particles, the mechanisms deployed to avoid explosion states, the quality of the Pseudo Random Number Generator (PRNGs) and the communication scheme adopted to exchange

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information among the particles. There are several works that tackle the three former issues [11, 8, 4]. The latter has been been widely discussed since it defines the neighborhood of the particles and, as a consequence, determines how the information flows through the whole swarm [6, 16].

Previous works have shown that less connected topologies slow down the information flow, since the information about the convergence is transmitted indirectly through intermediary particles [16]. On the other hand, highly connected topologies diminish the average distance between any pair of individuals. As a consequence, there is a tendency for the whole swarm to move quickly toward the first local optimum found by any particle of the swarm when the average distance between nodes is too short (*e.g.* a small-world topology). Unfortunately, in simple and static communication schema, fast convergence generally means premature convergence to a local optimum, specially in multimodal search spaces [6].

Recently, many efforts have been made to analyze how to link components in complex systems [17]. Some examples are social networks, World Wide Web, power grids [20] and biochemical networks [18]. In all these systems, there are several aspects that can be analyzed, such as the way these components can interact with themselves, or the pattern of connections between the components, which is in general highly correlated with the system behavior.

Until the last decades, perhaps due to the lack of deeper analysis or because of the limited processing capacity of computer, real-world networks were usually seen as a result of a completely random process [2]. Indeed, the study of real networks has gained relevance since they present many interesting features, such as fast spread of information through the network compounds, robustness, reliabilty [9, 10, 7].

Barabási and Albert showed that large real networks follow a scale-free power law distribution. They pointed out that this feature was a consequence of two underlying mechanism: (*i*) networks expand continuously by addition of new vertices; and (*ii*) new vertices usually attach to nodes that are already well connected [3]. Thus, they proposed a model, known as Barabási-Albert model (BA model), consisting of an algorithm for generating random scale-free networks using a preferential attachment mechanism [1]. A variation of the BA model, called Bianconi-Barabási model, that the probability of a node to connect to one another is given by a term that depends on the fitness of the involved node [5].

The idea of preferential attachment and complex networks was already proposed to define the PSO communication topology, as in the work of Godoy and von Zuben [13]. In this approach, the PSO starts with a scale-free topology generated by the BA model, and then, the particles are connected or disconnected along the iterations depending on the fitness of the particles. One may notice that there are some undesired outcomes from this approach: (*i*) since the swarm is initiated with a small-world topology, probably the swarm will present a high probability to be stuck in a local minima, specially in multimodal search spaces; (*ii*) the algorithm is not quick for connecting and disconnecting particles, and this behavior is not a desired feature for dynamic or multimodal problems; and (*iii*) the mechanism used to reconnect the particles does not take into account the past of the particle, it solely depends on the current fitness of the particle.

In this paper, we propose a novel approach to define the dynamic topology based on preferential attachment. The proposal aims to balance information flow in the swarm. The topology is initiated as a local topology and evolves to allow the particles to increase the communication capability when it is necessary. Besides, it also considers if the particles are improving or not their solutions. The paper is organized as follows: we briefly review the Particle Swarm Optimization in the next section. In Section 3, we present our proposal to define a dynamic communication topology. The simulation setup and results are given in Section 4. Finally, we present our conclusions and suggest some future works in the last section.

2 Particle Swarm Optimization

Particle Swarm Optimization (PSO) is composed by a swarm of particles, where each particle has a position within the search space $\mathbf{x_i}(t)$ and each position represents a possible solution for the optimization problem. The particles fly through the search space of the problem searching for the best solution. Each particle updates its position according to the current velocity $\mathbf{v}_i(t)$, the best position found by the particle itself [$\mathbf{P}_{best_i}(t)$] and the best position found by the neighborhood of the particle i during the search so far [$\mathbf{N}_{best_i}(t)$].

Therefore, the velocity and the position of every particle are updated iteratively by applying the following update equations for each particle in each dimension d:

$$\mathbf{v_i}(t+1) = \mathbf{v_i}(t) + r_1 c_1 [\mathbf{P}_{best_i}(t) - \mathbf{x_i}(t)] + r_2 c_2 [\mathbf{N}_{best_i}(t) - \mathbf{x_i}(t)], \tag{1}$$

$$\mathbf{x_i}(t+1) = \mathbf{x_i}(t) + \mathbf{v_i}(t+1), \tag{2}$$

where r_1 and r_2 are numbers randomly generated by an uniform distribution in the interval $[0,1]$. c_1 and c_2 are the cognitive and the social acceleration constants, respectively. The original PSO updates the velocities of the particles considering the current value for the velocity of the particles, as presented in equation (1). Clerc [8] performed a study on the dynamic of the particles and stated a parameter known as the constriction factor (χ) that avoids the explosion state. χ is defined in equation (3). The velocity update equation is depicted in equation (4).

$$\chi = \frac{2}{|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}|}, \ \varphi = c_1 + c_2, \tag{3}$$

$$\mathbf{v_i}(t+1) = \chi \cdot \{\mathbf{v_i}(t) + r_1 c_1 [\mathbf{P}_{best_i}(t) - \mathbf{x_i}(t)] + r_2 c_2 [\mathbf{N}_{best_i}(t) - \mathbf{x_i}(t)]\}. \tag{4}$$

The way the information flows through the particles is determined by the communication topology used by the swarm [12]. The topology of the swarm defines the neighborhood of each particle, that is the subset of particles which the particle is able to communicate with [6]. In the context of social networks, there are many factors that influence the flow of information between nodes [19, 20]. These aspects include the degree of connectivity among the nodes, the average number of neighbors in common per node and the average shortest distance between nodes.

Kennedy and Mendes analyzed these factors on the particle swarm optimization algorithm [16]. It has been shown that the presence of intermediaries slows the information flow down. On the other hand, the information moves faster if more pairs of individuals are connected. Thus, when the average distance between nodes are too short, there is a tendency for the population to move quickly toward the best solution found in earlier iterations. For simple unimodal problem, it usually implies in a faster convergence to the global optimum. However, this fast convergence might means a premature convergence to a local optimum, specially in multi-modal problems [6]. In this case, communication topologies with intermediaries, *i.e.* with a lower number of connections, could help to reach better results.

A first communication model proposed by [14] to model the natural behavior of flocks of birds presented a dynamic topology based on the distance between the particles. However, due to the high computational cost, it was discarded, albeit the similar behavior of flocks of birds [6]. The global topology, which is often known as $\mathbf{G}_{best}$, is a static topology proposed in the PSO white paper [15]. In the $\mathbf{G}_{best}$, all the particles of the swarm are neighbors of each particle of the swarm. This means that the social memory of the particles is shared by the entire swarm. This topology leads to a fast convergence, since the information spreads quickly. On the other hand, in less connected topologies, each particle only shares information with a subset of the swarm. Thus, the social memory is not the same for the whole swarm. The most used local topology is called $\mathbf{L}_{best}$. In the $\mathbf{L}_{best}$ approach, each particle has two neighbors and the neighbor is based on the index. For example, the neighbors of particle #2 are particles #1 and #3. The $\mathbf{L}_{best}$ helps to avoid a premature attraction of all particles to a single spot of the search space, once the information is spread slowly and the swarm has more chances to explore different regions of the search space. Nevertheless, it presents a slower convergence. The two extreme behaviors of the $\mathbf{G}_{best}$ and $\mathbf{L}_{best}$ topologies have encouraged efforts to propose approaches that can present fast convergence while avoiding local minima. Indeed, many other topologies were already proposed, such as *von Neumann*, *Focal*, *Four Clusters*, *Clan PSO*, among others.

Godoy and von Zuben proposed to use a scale-free based topology, called Complex Neighborhood based Particle Swarm Optimization (*CNPSO*)[13]. The evolution of the topology is based on the Barabási-Albert model and it tries to maintain the scale-free characteristic of the topology, while the optimization is being performed. In the *CNPSO*, the swarm topology starts with a scale-free topology generated by the BA-model and it does not take into account any particle information. Thus, it is possible to have a bad particle as a hub in the swarm. Moreover, the initial topology has a small mean-shortest path length. This feature is not desirable in the initial stages of the algorithm because it can attract the swarm to a local optimum in earlier iterations, since the information flows fast. The *CNPSO* reconnecting mechanism also does not take into account the fitness information through the iterations. For example, it does not matter if the particle has stagnated or not in a local optimum. After *times* number of iterations, random particles will have its connections mutated even if they are having success or not. Therefore, this approach is not dynamic in the sense that its mechanism is not based on the swarm condition but rather it is based on random particles in any state.

3 Our Proposal

We aim to create a dynamic topology which can balance the search behavior of the swarm. It begins with the swarm being less connected. As a consequence, the swarm will present a high capacity to explore along the entire search space. Besides, it is desirable to change the communication scheme of the particles as they reach a stagnation state. Thus, in order to state when a particle k is stagnated, a new attribute, named $P_k failures$, is included to the particles. If the particle k does not improve its position in the current iteration, $P_k failures$ is incremented, otherwise $P_k failures$ is set to zero.

As each particle tries to find better particles to be connected with, there is a preferential attachment connecting mechanism based on the particles fitness. Therefore, to have this mechanism, we used a roulette wheel based on a rank that depends on the fitness of the particles. The best particles have more chances to be chosen for new connections. The proposed algorithm is shown in Algorithm 1.

Algorithm 1. Pseudocode of our proposal

```
Generate the neighborhood of particles with a ring topology
Initialize position, velocity and personal best position of the N particles
while stop criterion is not satisfied do
    for k = 1 to N do
        Update Particle k
        if Particle k improved its position then
            Update p_k best position vector
            p_k failures ← 0
        else
            p_k failures ← p_k failures + 1
        if p_k failures > failures_threshold then
            for n = 1 to N do
                A particle r is chosen by using a roulette wheel based on the rank of
                the particles
                if n = r and p_n is better than p_k then
                    Connect Particle n to Particle k
                else
                    Disconnect Particle n and Particle k
        Update Particle k
```

The algorithm begins with a ring topology with N particles. For all PSO iterations, each particle k has $P_k failures$ updated according to the fitness evolution. When the threshold of failures ($failures\_threshold$) is reached, the particle searches for better particles to follow. The selection of new neighbors is based on a roulette wheel with a fitness-based rank.

The threshold of failures is crucial to the algorithm performance. If it has a low value, the particles will easily try to reconnect. Otherwise, particles will maintain the previous behavior for a long time.

4 Simulation Setup and Results

We used four well-known benchmark functions to evaluate our proposal and compare it to the previous approaches [6]. The functions are used for minimization problems. Two of them are unimodals, Rosenbrock and Ackley, and two are multimodal, Rastrigin and Griewank. The global optimum of all of them is at $(0,...,0)$.

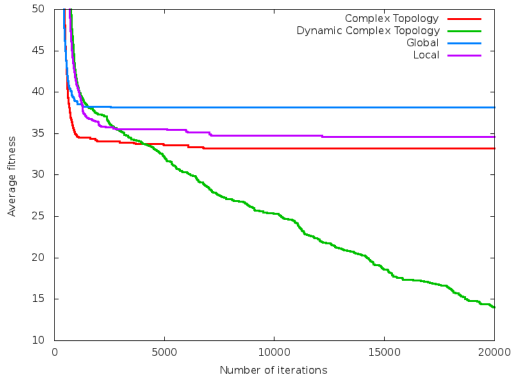
In all experiments, all functions were implemented in 30 dimensions. We have executed the PSO algorithm 30 times with $3,000$ iterations in all functions. The threshold of failures for the particles was set to 100. The particles were updated according to the Equation 4. We used $c_1 = 2.05$ and $c_2 = 2.05$.

Table 1 presents the mean value and the (standard deviation) of the best fitness found for each function by each tested topology. One can observe that the results achieved by our proposal are similar to the Local topology for the functions Ackley, Rosenbrock and Griewank, but we obtained the best performance for the Rastrigin function. One can also notice that we far outperformed the Global topology and the *CNPSO* approach (static complex topology).

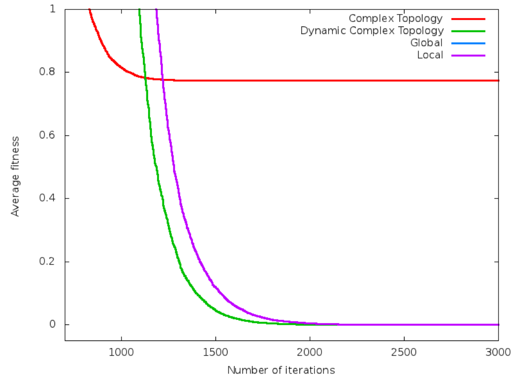
Table 1. Mean value and (standard deviation) of the best fitness found for each function

PSO Topology	Rastrigin	Ackley	Rosenbrock	Griewank
Global topology	38.1401	7.4857	0.0011	0.0134
	(9.2908)	(9.3576)	(0.0015)	(0.0189)
Local topology	34.5914	**0.0000**	**6.2587**$\times 10^{-8}$	**0.0025**
	(9.0085)	**(0.0000)**	**(1.6376**$\times 10^{-7}$**)**	**(0.0052)**
Static complex topology	33.1985	0.7740	0.0017	0.0119
	(8.6007)	(2.2623)	(0.0023)	(0.0148)
Our proposal	**14.0476**	**0.0000**	1.7766×10^{-7}	0.0037
(Dyn. Complex Topology)	**(5.2370)**	**(0.0000)**	(2.7928×10^{-7})	(0.0063)

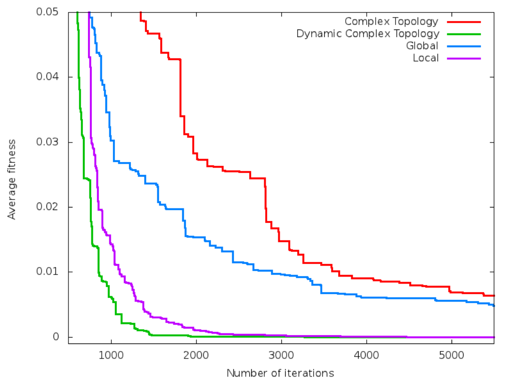
The average values of the best fitness achieved along the iterations by the PSO algorithm using the four different topologies for the functions Rastrigin, Ackley, Rosenbrock and Griewank are shown in Figure 2. As can be seen, our proposal converges faster for Rosenbrock and Ackley functions. Besides, our approach does not get stuck in local minima in the Rastrigin function, while all other tested approaches quickly stagnate.



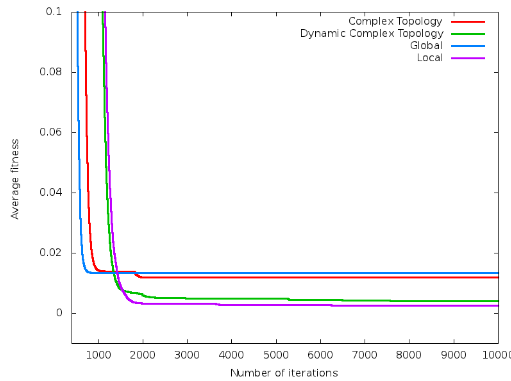
(a) Rastrigin



(b) Ackley



(c) Rosenbrock



(d) Griewank

Fig. 1. The average value of the best fitness achieved in each function through the iterations by the PSO algorithm using different topologies

5 Conclusions and Future Works

In this paper, a novel dynamic communication topology based on the Barabási-Albert model for the Particle Swarm Optimization is proposed. In this approach, the particles explore the search space at the beginning and, as the particles get stagnated, they try to seek for better particles to follow. This search for new neighbors is based on the preferential attachment of the Barabási-Albert model.

The simulation results showed that the proposed approach is in average better than other well known topologies and outperforms a simpler previously proposed topology based on the Barabási-Albert model.

For the future, we intend to test this approach in dynamic problems. We also intend to investigate the impact of the failures threshold of the particles in the optimization process and the impact of the initial topology as well.

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